Velocity fluctuations in electrostatically driven granular media

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We study experimentally the particle velocity fluctuations in an electrostatically driven dilute granular gas. The velocity distributions have strong deviations from a Maxwellian form over a wide range of parameters. We have found that the tails of the distribution functions are consistent with a stretched exponential law with typical exponents of the order 3/2. Molecular dynamic simulations shows qualitative agreement with experimental data. Our results suggest that this non-Gaussian behavior is typical of most inelastic gases with both short- and long-range interactions.

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Despite extensive study over the preceding decade, a fundamental understanding of the dynamics of granular materials still poses a challenge for physicists and engineers [1,2]. Driven granular materials exhibit complex behavior that resembles some aspects of conventional solids, liquids, and gases, yet there are some considerable differences [1–4].

Recent experimental studies with vibrationally driven granular gases revealed surprising deviations in the particle distribution function from the Maxwell distribution law [5–10]. These deviations were attributed to the effects of dissipation due to inelasticity of interparticle collisions. In particular, Ref. [5] reported an ''universal'' stretched exponential law

$$P(v) \sim \exp[-|v/v_0|^{\zeta}] \tag{1}$$

with the exponent $\zeta = 3/2$. Here v is the particle velocity, P is the velocity distribution function, and v_0 is the ''thermal velocity.'' This behavior was observed in a wide range of frequencies and amplitudes of vibration and for different densities of the granular gas and agrees with the theoretical prediction of Ref. [11]. Deviations from Maxwellian behavior have also been observed experimentally in different geometries and for different driving conditions in Refs. [7–9] and in numerical simulations [12–14]. These studies suggest that the deviations from a Maxwell distribution are the result of short-range inelastic hard-core collisions between particles.

Interactions between particles often are not reduced to simple hard-core collisions. Fascinating collective behavior appears when small particles acquire an electric charge and respond to competing long-range electromagnetic and short-range contact forces. An important question is whether non-Maxwellian distributions are typical only of granular gases with hard-core collisions or could they also be observed for general dissipative gases with both short-range and long-range interactions.

The electrostatic excitation of granular media offers unique new opportunities compared to traditional vibration techniques that have been developed to explore granular dynamics. It enables one to deal with extremely fine powders which are not easily controlled by mechanical methods. Electrostatic driving makes use of these *bulk* forces, and allows control of the ratio between long-range electric forces and short-range collisions by changing the amplitude and the frequency of the applied electric field. Our previous studies with electrostatically driven granular media revealed a hysteretic phase transition from the immobile condensed state (granular solid) to a fluidized dilated state (granular gas) with a changing applied electric field [15]. A spontaneous precipitation of dense clusters from the gas phase and a subsequent coarsening—coagulation of these clusters—is observed in a certain region of the electric field values. The strong effect of humidity on dynamics of electrostatically driven granular materials was studied in Ref. [16].

We study the particle velocity distributions in electrostatically driven granular media. We have found that in a wide range of parameters the particle velocity distribution function is strongly non-Maxwellian and is well approximated by the stretched exponential law $P(v) \sim \exp(-|v/v_0|^{3/2})$. We performed molecular dynamics simulations of conducting particles in an ac electric field and have obtained qualitative agreement with the experiment. We conclude from our results that the tails of the velocity distributions for driven granular gases, in general, exhibit non-Maxwellian behavior and are not limited to the systems with just hard-core collisions.

Our experimental setup is similar to that in Refs. [15,16]. Particles are placed between the plates of a large capacitor which is energized by a constant (dc) or alternating (ac) electric field $E = E_0 \cos(\omega t)$, see Fig. 1. To provide optical access to the cell, the capacitor plates were made of glass with a clear conductive coating. We used 11×11 cm capacitor plates with a spacing of 1.5 mm. The particles consisted of 165 μm diameter conducting bronze spheres. The field amplitude E_0 varied from 0 to 10 kV/cm and the frequencies $f = \omega/2\pi$ on the interval of 0 to 120 Hz. The total number of particles in the cell varied between 10^5 and 10^6.

Conducting particles acquire a surface charge when they are in contact with the capacitor plate. As the magnitude of the electric field in the capacitor exceeds a critical value, E_1, the upward electric force overcomes gravity mg (m is the mass of the particle, g is the gravity) and lifts the charged particles. When grains hit the upper plate, they deposit their charge and fall back. By applying an alternating electric field $E = E_0 \sin(2\pi f t)$, and adjusting its frequency f, one can con-

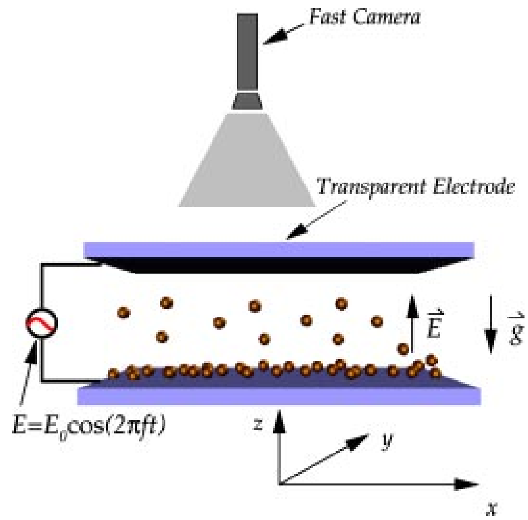

FIG. 1. Block diagram of the experimental apparatus.

trol the vertical excursion of particles by effectively turning them back before they collide with the upper plate. Thus, by increasing the frequency of the electric field, the particles can be confined in a relatively thin layer, i.e., the granular gas is quasi-two-dimensional at high frequencies and three-dimensional at low frequencies. It effectively allows for control over the number of collisions and, therefore, the contributions from long-range and short-range interactions. This control over the vertical extent of the motion of the particles also changes the manner in which energy is transferred from the vertical to the horizontal through collisions. From the comparison of typical kinetic energy acquired by the particle and electric energy of interparticle interaction it is possible to show that the long-range electric interaction becomes dominant with respect to short-range collisions at frequencies larger than $f_0 \approx 100$ Hz for a given particle size.

In our experiment we extracted horizontal particle velocities via high-speed image analysis. Pictures were obtained at the rate of 1000 frames per second from a camera mounted on a microscope suspended vertically above the cell and particle positions were resolved to subpixel resolution. Interparticle and particle-boundary collisions that introduce sudden changes in momenta of the particles were filtered from the distributions in a manner similar to that employed in Refs. [6,7]. These events were only a small fraction of the total number of measurements due to the low density at which they were acquired. An ensemble average for each of the velocity distributions was obtained with nearly 10^6 data points.

The effect of long-range electric forces is illustrated in Fig. 2. In contrast to vibrationally driven systems, the momentum transfer occurs often without actual collision. While there are hard-sphere collisions in the system as well, this interaction occurs at a center to center distance of approximately two particle diameters. The effect of long-range interactions is manifested also in curved particle trajectories in Fig. 2.

A summary of experimental results is presented in Figs. 3 and 4. We find for various values of frequency f and electric field amplitude E_0 that the normalized velocity distributions,

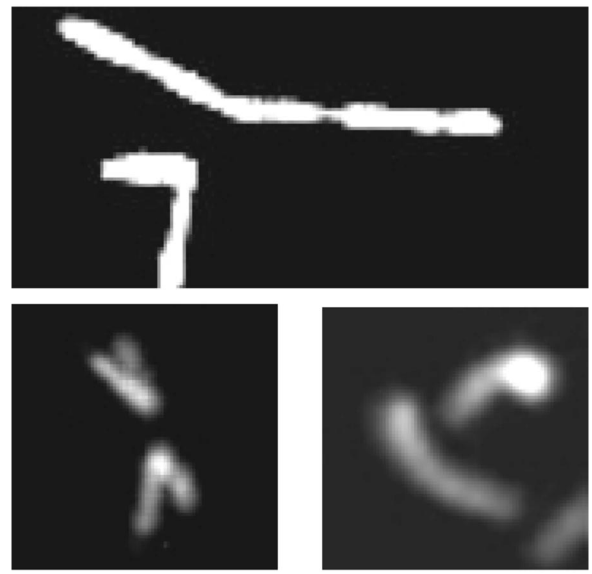

FIG. 2. Composite images of two particle collision created by overlaying several sequential images taken with the high-speed camera for a driving frequency of 50 Hz. Each of the three images demonstrates a different collision between two particles that occurs at a distance larger than a ball diameter. At this relatively high frequency the particles have a small vertical excursion and the focal plane of the microscope is set between the two plates of the cell. The particles are completely illuminated, which results in the smudged streaks of varying intensity in the particle tracks.

both in logarithmic and linear scales, lay practically on a top of each other (see Fig. 3). There is a very small difference in granular temperature T_g between x and y directions, which is probably related to some small tilt of the cell with respect to gravity. There is an increase in the flatness (kurtosis) of the velocity distributions, $F = \langle v^4 \rangle / \langle v^2 \rangle^2 - 3$, as the driving fre-

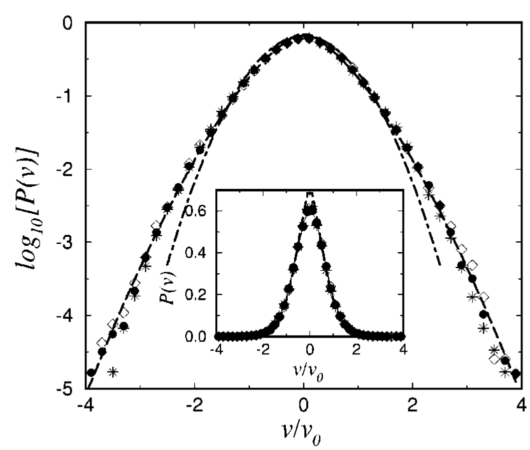

FIG. 3. Main plot: the logarithms of horizontal velocity distribution $P(v)$ vs normalized velocity v/v_0, where $v_0 = \sqrt{2T_g}$, T_g is horizontal granular temperature. Bullets correspond to $f=50$ Hz and $E_0 = 8.83$ kV/cm, stars to $f=120$ Hz and $E_0=8.83$ kV/cm, diamonds to $f=45$ Hz and $E_0=7.3$ kV/cm. The dashed line shows the best fit according to Eq. (1) with $\zeta \approx 1.51$ and the dot-dashed line shows the Gaussian fit. Inset: the horizontal velocity distribution $P(v)$ vs normalized velocity v/v_0 in linear scale.

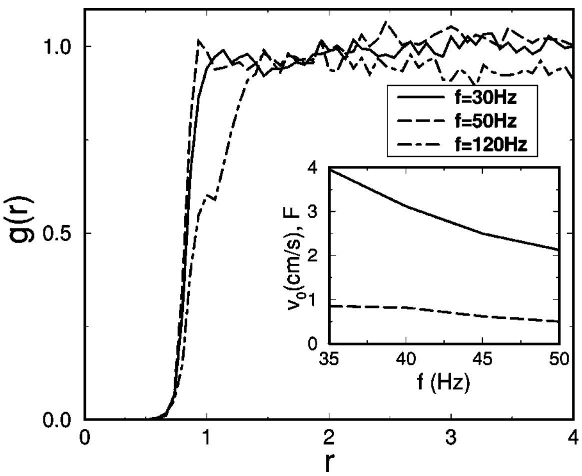

FIG. 4. The density distribution function $g(r)$ for different values of *f*. Inset: "thermal velocity" v_0 (solid line) and flatness F (dashed line) vs *f* for $E_0 = 8.83$ kV/cm.

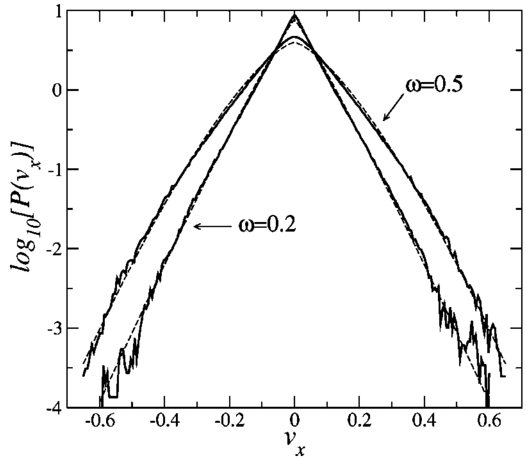

FIG. 5. Simulation results: the logarithms of horizontal velocity distribution $P(v_x)$ (x component only) for two different values of frequency ω. Solid lines show numerical results; dashed lines are fits $\log P(v) = a_0 - a_1 v^{\zeta}$, with $\zeta \approx 1.43$ for $\omega = 0.5$ and $\zeta \approx 1.08$ for $\omega = 0.2$.

quency is decreased at a fixed driving amplitude, see Fig. 4, inset. (As defined here, a flatness of 0 corresponds to a Gaussian distribution.) In a mechanically shaken granular layer, an increase in flatness was observed as the shaking amplitude was decreased at constant frequency, which corresponded to a lower horizontal granular temperature [7]. In the electrostatically driven system, the increase in flatness occurs as the horizontal granular temperature increases, as there is more kinetic energy to transfer from the horizontal to the vertical direction because of the increase in vertical excursion at lower frequencies.

Figure 4 shows the density distribution function $g(r) = \langle \rho(r)/\rho(0) \rangle / \langle \rho \rangle^2$ versus frequency. The volume density for the experimental conditions is extremely dilute ($\approx 7\%$), as can be seen by the lack of particle correlations with the exception of the excluded volume of the finite particles. The function $g(r)$ has almost steplike form for low frequencies. One can also see that with the increase of frequency the contribution from long-range interactions also increases. This fact is manifested in the smearing of the steplike structure of $g(r)$, which can be interpreted as an increase of the effective cross section for particle interactions. Moreover, the gradual decay of the flatness with the increase of frequency suggests crossover to Maxwellian velocity distribution for very high frequency when the long-range electromagnetic interactions dominate short-range collisions.

We performed molecular dynamics simulations of conducting particles in an applied ac electric field. We modified the MD code used in our earlier paper [15]. In particular, we introduced a finite roughness of the bottom plate (to simulate also the nonsphericity of particles on qualitative level) and finite restitution coefficient $\bar{r}$ between the particles and particles and walls ($\bar{r} = 0.8$ and $\bar{r} = 0.6$, correspondingly) [17]. We changed the restitution coefficient and roughness over a wide range, but the results appear to be very robust to the specific choice of $\bar{r}$. We simulated up to 1000 particles in the cell 80×80 particle diameters in the x-y plane, electric field amplitude E_0 was $2.6 E_1$, where E_1 is the first critical field value (for $E = E_1$ upward, electric force acting on an isolated particle overcomes gravity, see Ref. [15]). The velocity is scaled by $\bar{v} = \sqrt{ga}$ and time by $\bar{t} = \sqrt{a/g}$, where a is the radius of the particle. Typical simulation time was 10 000 $\bar{t}$, and the distributions were built from 2 500 000 sample points.

Selected results are presented in Fig. 5. In the wide range of parameters ω, E_0 we have found non-Maxwellian velocity distributions. For relatively high frequencies $\omega = 0.5$ (corresponding approximately to $f = 30$ Hz for our experimental parameters) the velocity distribution is indeed approximated by Eq. (1) with ζ close to 3/2, as in the experiment. For a smaller frequency ($\omega = 0.2$) we have found that $P(v)$ is consistent with a simple exponential law: $P(v) \sim \exp(-v/v_0)$, i.e., $\zeta = 1$. It is plausible that the crossover to exponent $\zeta = 3/2$ occurs for the high velocity tail and would explain the gentle increase in flatness as the frequency is reduced. However, one likely needs substantially longer simulations in order to resolve this crossover.

Our simulation results for "high frequency" yield an exponent $\zeta \approx 1.5$ and are in a good agreement with experiment. Unfortunately, we cannot verify the "low frequency" exponent $\zeta = 1$ because our current experimental apparatus does not allow us to extract reliable particle positions for the frequencies f below 30 Hz. For small frequencies the vertical particle displacements are much higher than the particle size. As a result, the particles go out of the focal plane of the microscope and it is impossible to sufficiently resolve horizontal positions.

Let us briefly discuss the results. Universal high velocity tails of the distribution $P \exp(-|v/v_0|^{3/2})$ were derived by van Noije and Ernst [11] from the Enskog-Boltzmann equation for the uniformly heated state of a two-dimensional weakly inelastic granular gas. According to Ref. [11], in the steady state, the averaged distribution function satisfies the equation

$$D\partial_v^2 P = -I(P,P), \tag{2}$$

where $I(P,P)$ is the pair collision integral and D is proportional to the strength of external ''thermal noise.'' In the large velocity limit Ref. [11] derived $I(P,P) \sim vP$ since the fast particle typically collides with particles in the ''thermal range,'' which immediately results in Eq. (1) for the high velocity tail.

Although there is no ''thermal noise'' in our experiment, multiple collisions between particles, the finite roughness of the plates, the nonsphericity of particles, and long-range interactions result in sufficient randomization of particle velocities and presumably have an overall effect similar to thermal noise. The argument that the collision integral for high velocities is reduced to vP appears to be very robust and does not depend on the specifics of the model. Thus, we can expect the above stretched exponential velocity tails to be characteristic of our system as well.

The distribution $P \sim \exp(-v/v_0)$ was derived for a freely evolving granular gas [18]. One can speculate that our results for the small frequency have some fingerprints of a ''freely evolving gas'' because the particles spend significant time in the flights between plates and exhibit some sort of ''free cooling,'' whereas for the higher frequencies the particles are confined in a thin layer and collisions with the bottom plate occur more often.

In conclusion, we experimentally and numerically studied particle velocity distributions in an electrostatically driven granular media. We have found that in a wide range of experimental parameters the particle velocity distribution function is strongly non-Maxwellian and is well approximated by the stretched exponential law $P(v) \sim \exp(-|v/v_0|^{3/2})$. This is additional evidence that short-range forces contribute to non-Gaussian distributions and the presence of long-range interactions does not change this. Our results suggest that such velocity distributions should be generic for dissipative gases with both short- and long-range interactions.

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[1] H.M. Jaeger, S.R. Nagel, and R.P. Behringer, Phys. Today **49**, 32 (1996); Rev. Mod. Phys. **68**, 1259 (1996).
[2] L.P. Kadanoff, Rev. Mod. Phys. **71**, 435 (1999).
[3] P.G. de Gennes, Rev. Mod. Phys. **71**, S374 (1999).
[4] J. Rajchenbach, Adv. Phys. **49**, 229 (2000).
[5] F. Rouyer and N. Menon, Phys. Rev. Lett. **85**, 3676 (2000).
[6] J.S. Olafsen and J.S. Urbach, Phys. Rev. Lett. **81**, 4369 (1998).
[7] J.S. Olafsen and J.S. Urbach, Phys. Rev. E **60**, R2468 (1999).
[8] A. Kudrolli, M. Wolpert, and J.P. Gollub, Phys. Rev. Lett. **78**, 1383 (1997).
[9] W. Losert, D.G.W. Cooper, and J.P. Gollub, Phys. Rev. E **59**, 5855 (1999).
[10] D.L. Blair and A. Kudrolli, Phys. Rev. E **64**, 050301(R) (2001).
[11] T.P.C. van Noije and M.H. Ernst, Granular Matter **1**, 58 (1998).
[12] J.A. Carrillo, C. Cercignani, and I.M. Gamba, Phys. Rev. E **62**, 7700 (2000).
[13] Sung Joon Moon, M.D. Shattuck, and J.B. Swift, Phys. Rev. E **64**, 031303 (2001).
[14] R. Soto, J. Piasecki, and M. Mareschal, Phys. Rev. E **64**, 031306 (2001).
[15] I.S. Aranson, D. Blair, V.A. Kalatsky, G.W. Crabtree, W.-K. Kwok, V.M. Vinokur, and U. Welp, Phys. Rev. Lett. **84**, 3306 (2000).
[16] D.W. Howell, I.S. Aronson, and G.W. Crabtree, Phys. Rev. E **63**, 050301 (2001).
[17] In contrast to the mechanical system where the effects of surface roughness are negligible due to large size of the particles (1 mm and more) in our system the plate roughness and particles nonsphericity are more important.
[18] S.E. Esipov and T. Pöschel, J. Stat. Phys. **86**, 1385 (1997).